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BLANK FORMS IN ALGEBRA.*

By Albert Harry Wheeler.

Originals in geometry are, perhaps, the best means to be had by the teacher for determining the grasp the pupils have on the subject and for teaching geometry in the true sense.

Originals in algebra are not commonly recognized as a class of examples by themselves, and the great bulk of the work done in the subject is by the use of ready made text-book problems.

The writer has been developing a method and means for presenting certain topics in such a way that all of the pupils are obliged to do some original work, and the present article, written at the request of the editors of The Mathematics Teacher, is illustrated by material which has been found to have value in the class-room both with beginners and with advanced students.

Process examples alone will be considered. It will be recognized that applied problems may easily be treated in a similar manner, and are, in fact, dealt with by means of problems of local interest, constructed by pupils who contribute facts from their own experience, or which relate to their neighborhood or to industries of their town. The value of such a treatment of applied problems is well known and widely recognized. It is commonly used by teachers with great success and much has been published in that line, but there remains a field in which very little has been accomplished but in which there is opportunity for much to be done. When once the value is recognized it is bound to be widely used for teaching mathematics and for creating a new interest in the operations of the science. If imperfectly understood, mathematical operations always create in the mind of the pupil a growing dislike for the subject which prevents sincere effort toward the mastery of the difficulties.

Process examples present to the pupil difficulties both of values and of operations.

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By removing from the statement of a process example all reference to values, a new type of exercise in mathematics is created in which operations alone are indicated. All elements of confusion due to the presence of particular values are avoided.

We will call such a combination of symbols of operation a blank form.

A blank form to suggest the difference of two squares may be written as follows:

$$(\ \ )^2 - (\ \ )^2.$$

When such a blank form is first presented to a class it does not take long for some one to suggest an answer in blank form such as:

$$[(\ \ ) + (\ \ )]\,[(\ \ ) - (\ \ )].$$

An identity in blank form may then be constructed which shows at a glance the essential structure of the difference of two squares expressed as the product of the sum of two numbers multiplied by the difference

$$(\ \ )^2 - (\ \ )^2 \equiv [(\ \ ) + (\ \ )]\,[(\ \ ) - (\ \ )].$$

A discussing with the class of the operations involved soon shows the teacher whether or not they know what to do and how to go about the work when special values are substituted in the blanks.

The next step is to require the class to fill in the original blank form in a great variety of ways,

$$(\ \ )^2 - (\ \ )^2,$$

and to have examples arising in this way solved by other members of the class.

In this way such process examples as the following will be immediately volunteered by the class:

$$x^2 - 4;\ y^2 - 9;\ z^2 - 25;\ 16 - a^2;\ 36b^2 - c^2.$$

When someone volunteers $x^2 - 3$ the difficulty can be immediately attended to and a special prescription adapted to the sufferer in question can be quickly compounded on the spot.

It is natural for the pupils to like to contribute something of

their own and considerable competition develops among the members of the class with everybody trying to invent something new and all hands doing something.

In this way a great variety of examples will be constructed in a few moments and weaknesses on the parts of individuals can be quickly discovered and overcome. The grasp on the whole subject will be strengthened and everyone will be ready and eager for something harder in the line of ready made book problems. They will be capable of doing much more difficult work because they have analyzed the operations and the structure of the examples which they are studying.

It will be found to be interesting to discuss with the class the reason why one of the two blank forms

$$(\quad)^4 - (\quad)^2 \text{ and } (\quad)^6 - (\quad)^3$$

has rational factors while the other does not, and to suggest changes which may be made so that both expressions or forms will appear as the difference of two squares.

Blank forms help in studying the reduction of fractions to lowest terms. Such blank forms as the following will be found to be valuable:

$$\frac{(\quad)}{(\quad) + (\quad)},$$

$$\frac{(\quad) + (\quad)}{(\quad)^2 - (\quad)^2}.$$

After the class has had considerable experience in simplifying fractional forms, the following blank form will be found to fix matters definitely in the mind:

$$\frac{(\quad + \quad)^2 - (\quad)^2}{(\quad)^2 - (\quad - \quad)^2}.$$

Sums and differences of like powers with all of the confusion incident thereto may be presented in a very simple manner by using blank forms such as

$$x^3 + (\quad);\; y^4 - (\quad);\; z^5 + (\quad).$$

The method applies readily and is especially adapted to irrational forms such as the following:

$$\frac{(\quad)}{\sqrt{(\quad)}};\qquad \frac{(\quad)}{\sqrt{(\quad)}};\qquad \frac{\sqrt[3]{(\quad)}}{\sqrt{(\quad)}};$$

$$\frac{(\quad)}{\sqrt{(\quad)+(\quad)}};\qquad \frac{(\quad)}{\sqrt{(\quad)}+(\quad)}.$$

The confusion which naturally exists in the minds of many pupils between the two last forms may be readily overcome by considering the operations without the distraction which commonly accompanies the presence of particular values.

Many other blank forms relating to rationalization may be invented such as:

$$\frac{\sqrt{(\quad)}}{\sqrt{(\quad)}+\sqrt{(\quad)}},\qquad \frac{\sqrt{(\quad)}+\sqrt{(\quad)}}{\sqrt{(\quad)}-\sqrt{(\quad)}}.$$

Partly filled-in blank forms are of great assistance in teaching such topics as trinomial squares.

For example,

$$x^2 + 6x + (\quad)$$

shows whether or not the pupil knows and understands the structure of the trinomial square. By leaving blank one or more of the terms of a trinomial square, it is possible to cause the pupil to examine expressions critically for the necessary and sufficient conditions to be satisfied in order that an expression shall be the square of a binomial.

Thus, such a blank form as

$$(\quad)^2 - 24x + 9$$

is a good test of the ability of the pupil to recognize the essentials of the form considered as being the square of a binomial.

Of course it will be recognized that many indeterminate blank forms may be presented, such as,

$$(\quad)^2 + 48xy + (\quad)^2.$$

Such a form as this arouses considerable competition among the members of the class in giving various combinations of numbers which may be substituted, and in discussing the results obtained.

Special trinomial forms are readily studied by means of partly filled-in blank forms.

The blank form

$$(\quad)^2 + 5(\quad) + 6$$

will be found to be a good introduction for

$$(a+b)^2 + 5(a+b) + 6,$$

and for others such as

$$(x-y)^2 + 5(x-y) + 6.$$

Indeterminate partly filled-in blank forms such as

$$m^2 + (\quad)m + 20$$

used in factoring will serve to fix the attention of the class on the last term, 20, and will require them to separate 20 into different pairs of factors and to use the proper sums for the coefficient of the second term.

Thus, the attention of the class is necessarily directed to the term which should be first examined in the process of factoring by inspection.

Other blank forms may be used, such for example as

$$a^2 + 6a + (\quad)$$

and

$$b^2 - 7b - (\quad).$$

It will be found to be interesting to discuss with the class the reasons for the possibility of finding a greater number of substitutions which can be used with one of these forms than with the other.

When studying polynomial expressions of the third degree of the form

$$x^3 + ax^2 + bx + c$$

the value of the blank forms will be seen by such preparatory material as

$$a^3 + 14a^2 + (\quad)a + 40,$$
$$b^3 + 13b^2 + (\quad)b + 36,$$
$$c^3 + (\quad)c^2 + (\quad)c - 48,$$
$$d^3 - (\quad)d^2 - (\quad)d - 30.$$

Someone will discover that the set of three factors 8, 5 and 1

and also 2, 2 and 10 of 40 have the same sum, 14, while the sums of the products taken two at a time are different, so that there is an ambiguity and the first form as given is indeterminate.

Then someone will discover the two different substitutions which can be made in the second illustration.

Such examples will show the class the necessity of examining an expression carefully before deciding on a result, and consequently they will see the danger of jumping at conclusions before they have critically examined an expression given them.

Mathematics must train judgment to be of value and mere machine like accuracy is dangerous if not accompanied by careful planning before carrying out some particular operation.

Blank forms are designed to help plan the work. They call attention to the structure and mathematical form.

They are designed to check the natural tendency to start something and then trust to luck that the answer will come somehow.

Many other blank forms may be used to help in the study of polynomial expressions and it will be found without exception that they serve to fix attention on the essentials of the algebraic forms of the topics studied. No class can work with such material without gaining a grasp on the subject which is impossible to some, if not to all, without some means.

Partly filled-in blank forms are of great value in such topics as fractions.

Thus, the fraction

$$\frac{c^2 + 7c + 12}{c^2 + (\quad)c + (\quad)}$$

when given to the class admits of a variety of substitutions.

The different results obtained should be discussed with the class until many substitutions have been given which admit of simplifying the resulting fraction.

It will be found to be difficult to find a better means at the command of the teacher for insuring the grasp of the principle studied.

Members of a class who possess a little ingenuity will produce a blank form as an answer for the result obtained by performing the operations

$$[\sqrt{(\quad)} + \sqrt{(\quad)}]^2.$$

The result may appear as

$$(\quad) + 2\sqrt{(\quad)(\quad)},$$

and someone may suggest the following and give a good reason for it:

$$(\quad) + 2(\quad)\sqrt{(\quad)(\quad)}.$$

A great variety of definite numerical or literal results may be obtained by substituting different values in the above blank form. Thus we may expect from the class such results as:

$$(\sqrt{5} + \sqrt{2})^2,$$
$$(\sqrt{x} + \sqrt{y})^2,$$
$$(\sqrt{2a} + \sqrt{3a})^2.$$

The drill obtained in working these examples will afford a great amount of practice in the class-room and it will become very evident as to just who means business and who does not.

When someone in the class suggests

$$(\sqrt{3} + \sqrt{-3})^2$$

an interesting situation develops which gives the teacher an opportunity to enter the very interesting field of imaginary numbers.

By using blank forms it is possible to present a definite principle or problem to a class, and by holding back one or more of the conditions or elements in the statement of the question, by using blank forms to be filled in, it is possible to hold in check those members of the class whose reaction time in mathematics is short, and who always respond quickly, while others, whose reaction time is longer, become confused and discouraged, because by the time they have grasped the meaning of the question its value has been destroyed for them through its solution by someone else.

Blank forms enable the teacher to ask, "What are you going to do when you know all of the facts?"

Furthermore, they serve to make the class keen to get all of the necessary facts before beginning to operate.

When studying irrational equations blank forms will be found to be very interesting.

Such a group of suggested irrational equations as the following will help in the matter of extra roots:

$$\begin{aligned}\sqrt{(\quad)} + \sqrt{(\quad)} + \sqrt{(\quad)} &= 0,\\ \sqrt{(\quad)} - \sqrt{(\quad)} + \sqrt{(\quad)} &= 0,\\ \sqrt{(\quad)} + \sqrt{(\quad)} - \sqrt{(\quad)} &= 0,\\ \sqrt{(\quad)} - \sqrt{(\quad)} - \sqrt{(\quad)} &= 0.\end{aligned}$$

Such a set of equations when filled in with the same numbers and letters in the same columns will help in showing that one of the two values obtained by solving the resulting equations sometimes satisfies one of the given equations; another value often satisfies some other equation; perhaps both values satisfy a third and neither satisfies the remaining equation.

An identity, partly in blank form, will often be of value in studying polynomials as an introduction to certain relations in the theory of equations.

For example,

$$(x + (\quad))(x + (\quad))(x + (\quad)) \equiv x^3 + (\quad)x^2 + (\quad)x + (\quad)$$

will help, both in factoring and in the theory of equations.

The relations existing between the roots and coefficients of an equation are readily brought out by a set of equations which are partly in blank form, such as,

$$\begin{aligned}a^2 + (\quad)a + (\quad) &= 0,\\ b^2 - (\quad)b + (\quad) &= 0,\\ c^2 + (\quad)c - (\quad) &= 0.\end{aligned}$$

The class may be asked to find, if possible, some other blank form and to determine the signs for another equation suggested by

$$d^2 \quad (\quad)d \quad (\quad) = 0.$$

When used to their best advantage blank forms enable the teacher to show what can be done and why it should be done. They train the pupils to try to discover what should be done first. They tend to encourage the careful examination of a problem proposed and the pupils using them will be better qualified to

discriminate between different ways for securing a result and to choose the better way. They encourage originality, and above all, they show the teacher the way the mind of the pupil is acting and they reveal his purpose as to what he proposed to do, and an opportunity is often gained to head off the wrong method and thus to prevent the student from learning the lesson in the wrong way.

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